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THERMAL DIAGNOSTICS OF FRICTION IN CYLINDRICAL CONNECTIONS. I. ALGORITHM OF THE ITERATION SOLUTION OF AN INVERSE BOUNDARY-VALUE PROBLEM

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An algorithm for restoration of the intensity of heat release that results from the friction of cylindrical connections is suggested. The thermal diagnostics of friction in cylindrical connections is considered with the example of identification of the friction power in sliding bearings made of a polymer composite material from temperature measurements.

An analysis of works concerned with investigation of the structure of the energy balance of friction shows that the prevailing part (more than 85%) of the mechanical energy consumed by friction is converted into heat. The energy intensity of other components of the conversion process is low as compared to the generated heat [1, 2]. Thus, the intensity of heat release in the zone of frictional contact correlates with the value of the friction power. In cylindrical connections, the friction power is characterized by the moment of frictional force, whose direct measurement is carried out with the aid of different torsiometers and tensobeams. It is practically impossible to place the latter in the friction units of the facilities used, which substantially decreases the information content of the tests. In [3, 4], a method of thermal diagnostics of friction that allows restoration of the moment of frictional force in a sliding bearing from a temperature measurement at one point of an immovable element at a known distance from the contact zone is suggested and experimentally tested. In this method, it has been assumed that the thermophysical characteristics are independent of temperature and the corresponding linear inverse boundary-value problem is solved.

In the present work, based on the method of iteration regularization [5], we construct an algorithm of restoration of the moment of frictional force in a sliding bearing from temperature data with account for the dependence of the thermophysical characteristics on temperature.

We use a two-dimensional formulation of the thermal problem for a sliding bearing [3]. A schematic of the bearing is shown in Fig. 1. Sliding occurs over the surface of contact between elements 1 and 2; the bushing is rigidly connected to the bearing race. The shaft 1 and the race 3 are made of a metal, while the bushing is manufactured from a polymer composite material. The thermophysical parameters for the materials of the shaft, the bushing, and the race will be denoted by indices 1, 2, and 3, respectively.

We adopt the following main assumptions: 1) heat releases on the surface of contact between the shaft and the bushing;

2) the rotational speed of the shaft is considered to be rather high (more than 5 rad/sec), which allows us to assume that the heat flux at the boundary is uniformly distributed over the shaft circumference;

3) considering the significant difference of the thermal conductivities of the metallic and polymeric elements of the friction unit (up to two orders of magnitude), we assume that the temperature field in the shaft cross section is homogeneous.

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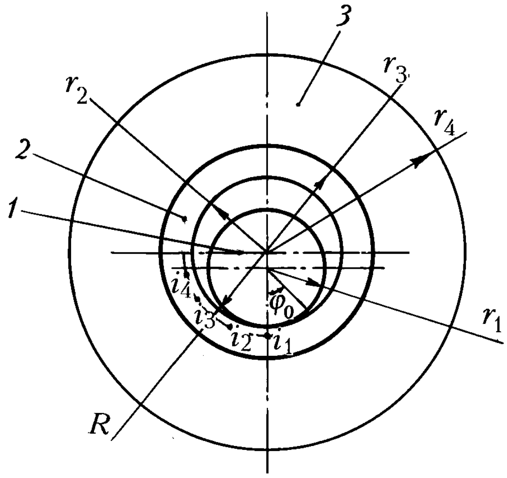

Fig. 1. Schematic of a friction unit: 1, shaft; 2, bushing; 3, race.

The temperature distribution in the bushing with the race is described by the two-dimensional equation

$$C_i(T)\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_i(T)\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left(\lambda_i(T)\frac{\partial T}{\partial\varphi}\right),\quad i=2,3,\quad r_2<r<r_3,$$

$$r_3<r<r_4,\quad 0<\varphi<\pi,\quad 0<t\le t_{\mathrm{m}},\tag{1}$$

with the initial condition

$$T(r,\varphi,0)=T_0.\tag{2}$$

In the contact zone for $|\varphi|\le\varphi_0$, we write the following condition of thermal conjugation [3]:

$$S_1C_1(U)\frac{dU(t)}{dt}+2(\pi-\varphi_0)\,r_1\alpha_1(U(t)-T_{\mathrm{a}})=Q(t)+2r_2\int_0^{\varphi_0}\lambda_2(T)\left.\frac{\partial T(r,\varphi,t)}{\partial r}\right|_{r=r_2}d\varphi,\tag{3}$$

$$T(r_2,\varphi,t)=U(t).\tag{4}$$

On the free surfaces of the bushing and the race, we prescribe the conditions of convective heat transfer

$$\lambda_2(T)\left.\frac{\partial T(r,\varphi,t)}{\partial r}\right|_{r=r_2}=\alpha_2(T(r_2,\varphi,t)-T_{\mathrm{a}}),\quad |\varphi|>\varphi_0,\tag{5}$$

$$\lambda_3(T)\left.\frac{\partial T(r,\varphi,t)}{\partial r}\right|_{r=r_4}=-\alpha_3(T(r_4,\varphi,t)-T_{\mathrm{a}}),\tag{6}$$

while at the boundary of the bushing and the race we assume the conditions of ideal thermal contact:

$$\lambda_2(T)\left.\frac{\partial T(r,\varphi,t)}{\partial r}\right|_{r=r_3^-}=\lambda_3(T)\left.\frac{\partial T(r,\varphi,t)}{\partial r}\right|_{r=r_3^+},\tag{7}$$

$$T(r_3^-, \varphi, t) = T(r_3^+, \varphi, t) \; . \tag{8}$$

Along the axis of application of a load, let the condition of symmetry of the temperature field be fulfilled:

$$\left.\frac{\partial T(r, \varphi, t)}{\partial \varphi}\right|_{\varphi=0} = \left.\frac{\partial T(r, \varphi, t)}{\partial \varphi}\right|_{\varphi=\pi} = 0 \; . \tag{9}$$

The inverse boundary-value problem of restoration of the moment of frictional force is formulated as follows: it is necessary to determine the function $Q(t)$ and the moment of frictional force $M(t)$ related by the formula $M(t) = Q(t)r_1/V$ from the system of equations (1)–(9) with the boundary and initial conditions for the known temperatures at the points of the bushing

$$T(R, \varphi_j, t) = f(\varphi_j, t) \; , \quad r_2 < R < r_3 \; , \quad 0 < \varphi_j \le \varphi_0 \; , \quad j = 1, ..., n \; .$$

The problem is solved by the method of iteration regularization. We consider an extremal formulation of the problem. As a measure of the deviation of the calculated temperatures $T(R, \varphi_j, t)$ from the measured $f(\varphi_j, t)$, we choose the root-mean-square discrepancy

$$J[Q(t)] = \sum_{j=1}^{n} \int_0^{t_{\mathrm{m}}} [T(R, \varphi_j, t) - f(\varphi_j, t)]^2 \, dt \; . \tag{10}$$

Then the inverse boundary-value problem is formulated as follows. We have to minimize functional (10) with constraints in the form of the system of equations (1)–(9). The function $Q(t)$ is the control quantity.

The distinctive feature of this problem is the presence of the nontraditional boundary condition in the friction zone, which imparts certain difficulties to constructing an algorithm of solution of the inverse problem.

The key problem in gradient minimization is determination of the gradient of the discrepancy functional (10), i.e., determination of the first Frechet derivative. The function $J'(t)$ is called the gradient of functional (10) at the instant t at the point $Q(t)$ if the increment in the functional can be represented in the form

$$J(Q + \Delta Q) - J(Q) = \int_0^{t_{\mathrm{m}}} J'(t) \, \Delta Q(t) \, dt + o(\|\Delta Q\|) \; . \tag{11}$$

An efficient method of determination of the gradient is the use of a conjugate boundary-value problem [5] whose derivation is presented below. Let us give the increment $\Delta Q(t)$ to the control function $Q(t)$; in this case, the temperature acquires the increment $\bar{v}(r, \varphi, t)$. Considering Eqs. (1)–(9) with the controls Q and $Q + \Delta Q$, we obtain the following system of equations for the temperature increment $\bar{v}(r, \varphi, t)$:

$$C_i \frac{\partial \bar{v}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_i \frac{\partial \bar{v}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\lambda_i \frac{\partial \bar{v}}{\partial \varphi}\right) + \lambda_i' \frac{\partial T}{\partial r}\frac{\partial \bar{v}}{\partial r} + \frac{\lambda_i'}{r^2}\frac{\partial T}{\partial \varphi}\frac{\partial \bar{v}}{\partial \varphi} - \left[C_i' \frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_i' \frac{\partial T}{\partial r}\right) - \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\lambda_i' \frac{\partial T}{\partial \varphi}\right)\right]\bar{v} \; , \tag{12}$$

$$r_2 < r < r_3 \; , \quad r_3 < r < r_4 \; , \quad 0 < \varphi < \pi \; , \quad 0 < t \le t_{\mathrm{m}} \; , \quad i = 2, 3 \; ;$$

$$\bar{v}(r, \varphi, 0) = 0 \; ; \tag{13}$$

$$S_1\left[C_1\frac{d\bar{v}_{sh}}{dt}+C_1'\frac{dU}{dt}\bar{v}_{sh}\right]+2(\pi-\varphi_0)\,r_1\alpha_1\bar{v}_{sh}=\Delta Q(t)+2r_2\int_0^{\varphi_0}\left(\lambda_2\frac{\partial\bar{v}}{\partial r}+\lambda_2'\frac{\partial T}{\partial r}\bar{v}\right)\Bigg|_{r=r_2}d\varphi\,,\quad |\varphi|<\varphi_0\,; \tag{14}$$

$$\bar{v}(r_2,\varphi,t)=\bar{v}_{sh}(t)\,,\quad |\varphi|<\varphi_0\,; \tag{15}$$

$$\lambda_2\frac{\partial\bar{v}}{\partial r}\bigg|_{r=r_2}=\left[\alpha_2-\lambda_2'\frac{\partial T}{\partial r}\bigg|_{r=r_2}\right]\bar{v}(r_2,\varphi,t)\,,\quad |\varphi|>\varphi_0\,; \tag{16}$$

$$\lambda_3\frac{\partial\bar{v}}{\partial r}\bigg|_{r=r_4}=-\left[\alpha_3+\lambda_3'\frac{\partial T}{\partial r}\bigg|_{r=r_4}\right]\bar{v}(r_4,\varphi,t)\,; \tag{17}$$

$$\left[\lambda_2\frac{\partial\bar{v}}{\partial r}+\lambda_2'\frac{\partial T}{\partial r}\bar{v}\right]\bigg|_{r=r_3^-}=\left[\lambda_3\frac{\partial\bar{v}}{\partial r}+\lambda_3'\frac{\partial T}{\partial r}\bar{v}\right]\bigg|_{r=r_3^+}\,; \tag{18}$$

$$\bar{v}(r_3^-,\varphi,t)=\bar{v}(r_3^+,\varphi,t)\,; \tag{19}$$

$$\frac{\partial\bar{v}}{\partial\varphi}\bigg|_{\varphi=0}=\frac{\partial\bar{v}}{\partial\varphi}\bigg|_{\varphi=\pi}=0\,, \tag{20}$$

where

$$\lambda_i'=\frac{d\lambda_i(T)}{dT}\,;\quad C_i'=\frac{dC_i(T)}{dT}\,.$$

Here, the linear part of the increment in functional (10) has the form

$$\Delta J=J(Q+\Delta Q)-J(Q)=2\sum_{j=1}^{n}\int_0^{t_m}[T(R,\varphi_j,t)-f(\varphi_j,t)]\,\bar{v}(R,\varphi_j,t)\,dt\,.$$

For functional (10) to acquire the extreme value, the following equality must be satisfied:

$$\Delta I=0\,, \tag{21}$$

where

$$I=I_0+I_1+I_2;$$

$$I_0=\Delta J=2\sum_{j=1}^{n}\int_0^{t_m}[T(R,\varphi_j,t)-f(\varphi_j,t)]\,\bar{v}(R,\varphi_j,t)\,dt\,;$$

$$I_1 = \int_{r_2}^{r_4}\int_0^{\pi}\int_0^{t_m} \left\{\left[-C_i\frac{\partial\bar{v}}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_i\frac{\partial\bar{v}}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left(\lambda_i\frac{\partial\bar{v}}{\partial\varphi}\right)\right]+\lambda_i'\frac{\partial T}{\partial r}\frac{\partial\bar{v}}{\partial r}+\right.$$

$$\left.+\frac{\lambda_i'}{r^2}\frac{\partial T}{\partial\varphi}\frac{\partial\bar{v}}{\partial\varphi}-\left[C_i'\frac{\partial T}{\partial t}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_i'\frac{\partial T}{\partial r}\right)-\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left(\lambda_i'\frac{\partial T}{\partial\varphi}\right)\right]\bar{v}\right\}\Psi\,(r,\varphi,t)\,dtd\varphi dr\,;$$

$$I_2 = \int_0^{\pi}\int_{r_2}^{r_4}\eta\,(r,\varphi,0)\,\bar{v}\,(r,\varphi,0)\,drd\varphi+\int_0^{\varphi_0}\int_0^{t_m}\left\{S\left(C_1\frac{d\,\bar{v}_{\rm sh}}{dt}+C_1'\frac{dU}{dt}\bar{v}_{\rm sh}\right)+\right.$$

$$\left.+2\,(\pi-\varphi_0)\,r_1\alpha_1\bar{v}_{\rm sh}\,(t)-2r_2\int_0^{\varphi_0}\left(\lambda_2\frac{\partial\bar{v}}{\partial r}+\lambda_2'\frac{\partial T}{\partial r}\bar{v}\right)\Big|_{r=r_2}d\varphi-\Delta Q\,(t)\right\}\times$$

$$\times\,\eta\,(r_2,\varphi,t)\,dtd\varphi+\int_0^{\varphi_0}\int_0^{t_m}[\bar{v}\,(r_2,\varphi,t)-\bar{v}_{\rm sh}\,(t)]\,\eta\,(r_2,\varphi,t)\,dtd\varphi+$$

$$+\int_{\varphi_0}^{\pi}\int_0^{t_m}\left\{\lambda_2\frac{\partial\bar{v}}{\partial r}-\left(\alpha_2-\lambda_2'\frac{\partial T}{\partial r}\right)\bar{v}\right\}\Big|_{r=r_2}\eta\,(r_2,\varphi,t)\,dtd\varphi+$$

$$+\int_0^{\pi}\int_0^{t_m}\left\{\lambda_3\frac{\partial\bar{v}}{\partial r}+\left(\alpha_3+\lambda_3'\frac{\partial T}{\partial r}\right)\bar{v}\right\}\Big|_{r=r_4}\eta\,(r_4,\varphi,t)\,dtd\varphi+$$

$$+\int_0^{\pi}\int_0^{t_m}\left\{\left(\lambda_2\frac{\partial\bar{v}}{\partial r}+\lambda_2'\frac{\partial T}{\partial r}\bar{v}\right)\Big|_{r=r_3^-}-\left(\lambda_3\frac{\partial\bar{v}}{\partial r}+\lambda_3'\frac{\partial T}{\partial r}\bar{v}\right)\Big|_{r=r_3^+}\right\}\eta\,(r_3,\varphi,t)\,dtd\varphi+$$

$$+\int_0^{\pi}\int_0^{t_m}\left(\bar{v}\,|_{r=r_3^-}-\bar{v}\,|_{r=r_3^+}\right)\eta\,(r_3,\varphi,t)\,dtd\varphi+\int_{r_2}^{r_4}\int_0^{t_m}\left\{\frac{\partial\bar{v}}{\partial\varphi}\Big|_{\varphi=0}\eta\,(r,0,t)-\frac{\partial\bar{v}}{\partial\varphi}\Big|_{\varphi=\pi}\eta\,(r,\pi,t)\right\}dtdr\,;$$

$\eta(r, \varphi, 0)$, $\eta(r_2, \varphi, t)$, $\eta(r_3, \varphi, t)$, $\eta(r_4, \varphi, t)$, $\eta(r, 0, t)$, $\eta(r, \pi, t)$, and $\Psi(r, \varphi, t)$ are the indeterminate Lagrange multipliers; ΔI is the total variation of the functional I.

Applying the basic lemma of variational calculus for the fulfillment of the steadiness condition (21) and equating each of the groups of summands in different variations to zero, we arrive at a system of equations relative to the Lagrange multipliers. Having eliminated the multipliers η from this system, we will have the conjugate boundary-value problem for determination of the functions $\Psi(r, \varphi, t)$ and $\Psi_{\rm sh}(t)$

$$-C_i\frac{\partial\Psi}{\partial t}=\frac{\partial}{\partial r}\left(r\lambda_i\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)\right)+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left(\lambda_i\frac{\partial\Psi}{\partial\varphi}\right)-$$

$$-r\lambda_i'\frac{\partial T}{\partial r}\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)-\frac{\lambda_i'}{r^2}\frac{\partial T}{\partial \varphi}\frac{\partial \Psi}{\partial \varphi}+2\sum_{j=1}^{n}\left[T(R,\varphi_j,t)-f(\varphi_j,t)\right]\bar{\delta}(r-R)\,\bar{\delta}(\varphi-\varphi_j)\,, \tag{22}$$

$$r_2<r<r_3\,,\quad r_3<r<r_4\,,\quad 0<\varphi<\pi\,,\quad 0<t\le t_{\mathrm{m}}\,,\quad i=2,3\,,$$

in which

$$\bar{\delta}(x)=\begin{cases}1\,, & x=0\,,\\ 0\,, & x\neq 0\,,\end{cases}\quad (r,\varphi,t_{\mathrm{m}})=0\,,$$

$$-S_1C_1\frac{d\Psi_{\mathrm{sh}}(t)}{dt}+2(\pi-\varphi_0)\,r_1\alpha_1\Psi_{\mathrm{sh}}(t)=2r_2\int_0^{\varphi_0}\left(r\lambda_2\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)\right)\Bigg|_{r=r_2}d\varphi\,,\quad |\varphi|\le\varphi_0\,,$$

$$(r_2,\varphi,t)=\Psi_{\mathrm{sh}}(t)\,,\quad |\varphi|\le\varphi_0\,,$$

$$\lambda_2\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)\Bigg|_{r=r_2}=\alpha_2\frac{\Psi}{r}\Bigg|_{r=r_2}\,,\quad |\varphi|>\varphi_0\,,$$

$$\lambda_3\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)\Bigg|_{r=r_4}=-\alpha_3\frac{\Psi}{r}\Bigg|_{r=r_4}\,,$$

$$\left(\lambda_2\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)\right)\Bigg|_{r=r_3^-}=\left(\lambda_3\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)\right)\Bigg|_{r=r_3^+}\,,$$

$$(r_3^-,\varphi,t)=\Psi(r_3^+,\varphi,t)\,,\qquad \frac{\partial\Psi}{\partial\varphi}\Bigg|_{\varphi=0}=\frac{\partial\Psi}{\partial\varphi}\Bigg|_{\varphi=\pi}=0\,.$$

Now we will derive a formula for determination of the gradient of functional (10). We express the linear part of the increment in the functional $J[Q(t)]$ from Eq. (22):

$$\Delta J=2\sum_{j=1}^{M}\int_{r_2}^{r_4}\int_0^{\pi}\int_0^{t_{\mathrm{m}}}\left[T(R,\varphi_j,t)-f(\varphi_j,t)\right]\bar{v}(r,\varphi,t)\,\bar{\delta}(R-r)\,\bar{\delta}(\varphi-\varphi_j)\,dtd\varphi dr=$$

$$=-\int_{r_2}^{r_4}\int_0^{\pi}\int_0^{t_{\mathrm{m}}}\left\{C_i\frac{\partial\Psi}{\partial t}+\frac{\partial}{\partial r}\left(r\lambda_i\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)\right)+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left(\lambda_i\frac{\partial\Psi}{\partial\varphi}\right)-r\lambda_i'\frac{\partial T}{\partial r}\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)-\frac{\lambda_i'}{r^2}\frac{\partial T}{\partial\varphi}\frac{\partial\Psi}{\partial\varphi}\right\}\bar{v}(r,\varphi,t)\,dtd\varphi dr\,. \tag{23}$$

Performing integration by parts, where necessary, using the equation for the increment $\bar{v}(r,\varphi,t)$ (12) and conditions (13), (16)–(20), we obtain

$$\Delta J=\int_0^{\varphi_0}\int_0^{t_{\mathrm{m}}}\left[r\lambda_2\frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right)\bar{v}-\lambda_2\frac{\partial\bar{v}}{\partial r}\Psi\right]\Bigg|_{r=r_2}dtd\varphi\,. \tag{24}$$

Whence, with account for the conditions in the contact zone (14), (15), we have

$$\Delta J = \int_0^{t_m} \frac{\Psi_{sh}(t)\,\Delta Q(t)}{r_2}\,dt\,. \tag{25}$$

Thus, the gradient of functional (10) is determined from the formula

$$J'(Q(t)) = \frac{\Psi_{sh}(t)}{r_2}\,. \tag{26}$$

To determine the gradient of the discrepancy functional with the known $Q(t)$, it is necessary to solve two boundary-value problems, namely, primal and conjugate. Solving the primal problem, we determine the nonstationary temperature field in a sliding bearing. Using the obtained temperature distribution $T(r, \varphi, t)$, we solve the conjugate problem and determine the functional gradient from (26), which allows us to restore the moment of frictional force in the sliding bearing by one of the gradient methods of functional minimization.

Minimization of functional (10) by the method of conjugate gradients can be represented by the following chain:

$$Q^k(t) \to T(r, \varphi, t) \to \Psi(r, \varphi, t) \to J'(Q^k) \to \gamma_k \to S^k(t) \to \bar{v}(r, \varphi, t) \to \beta_k \to Q^{k+1}(t)\,,$$

where

$$Q^{k+1}(t) = Q^k(t) - \beta_k S^k(t)\,,\quad k = 0, 1, ...,;\qquad S^k(t) = J'^k(t) + \gamma_k S^{k-1}(t)\,,\quad \gamma_0 = 0\,;$$

$$\beta_k = \frac{\sum_{j=1}^{n}\int_0^{t_m} [T(R, \varphi_j, t) - f(\varphi_j\, t)]\,\bar{v}(R, \varphi_j, t)\,dt}{\sum_{j=1}^{n}\int_0^{t_m} \bar{v}^2(R, \varphi_j, t)\,dt}\,;\quad \gamma_k = \frac{\int_0^{t_m}[\Psi^k(t)]^2\,dt}{\int_0^{t_m}[\Psi^{k-1}(t)]^2\,dt}\,.$$

The initial approximation of $Q^0(t)$ is prescribed; at each step $S^k(t)$ is used instead of $\Delta Q(t)$.

The efficiency of using the gradient methods in solution of different inverse problems is shown in [5]. Thus, the algorithm for restoration of the intensity of heat release due to friction and, correspondingly, of the friction power with account for the dependence of the thermophysical characteristics on temperature can be considered to be constructed.

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NOTATION

Q and ΔQ, intensity of heat release and its increment in the zone of frictional contact; M, moment of frictional force; V, slip velocity; T, bearing temperature at the initial instant of time; T, $\bar{v}$, bearing temperature and its increment; T_a, ambient temperature; U and $\bar{\upsilon}_{sh}$, averaged shaft temperature and its increment; t, running time; r, φ, polar coordinates; φ_0, half-angle of contact of the shaft with the bushing; i_k, points of a temperature measurement for a fixed radius R; t_m, test time; S_1, cross-sectional area of the shaft; $C_i (i = 1, 2, 3)$, volumetric heat capacity; λ_i $(i = 1, 2, 3)$, thermal conductivity; α_i $(i = 1, 2, 3)$, coefficient of heat transfer from the surface of the shaft and the free surfaces of the bushing and the race; η, Ψ, and Ψ_{sh}, Lagrange multipliers; S^k, conjugate direction; β_k, depth of descent; γ_k, iteration coefficient.

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